Math 141

Midterm 1

February 28, 2019

Name: Solutions

UR ID: ______________________

Circle your Instructor's Name:

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PLEASE COPY THE HONOR PLEDGE AND SIGN:

I affirm that I will not give or receive any unauthorized help on this exam, and all work will be my own.

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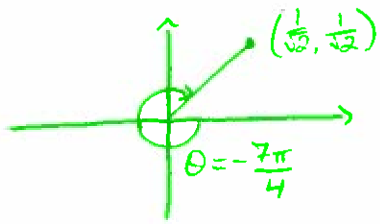
YOUR SIGNATURE:______________________

1. (8 points) Find the exact values:

(a) $\cos(-7\pi/4)$



Answer: $\frac{1}{\sqrt{2}}$ $\left(\text{or } \frac{\sqrt{2}}{2}\right)$

(b) $\tan(\pi/6)$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

Answer: $\frac{1}{\sqrt{3}}$ $\left(\text{or } \frac{\sqrt{3}}{3}\right)$

(c) $(\sin^{-1} \circ \sin)(\pi/3)$

$(\sin^{-1} \circ \sin)(\pi/3) = \frac{\pi}{3}$ since $-\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2}$

Answer: $\frac{\pi}{3}$

(d) $(\sin^{-1} \circ \sin)(2\pi/3)$

$$\sin^{-1}\left(\sin\left(\frac{2\pi}{3}\right)\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$$

since $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ and $-\frac{\pi}{2} \leq \frac{\pi}{3} \leq \frac{\pi}{2}$.

Answer: $\frac{\pi}{3}$

2. (8 points) Solve for θ in the interval $[0, 2\pi)$:

$$2\sin^2\theta - \sin\theta = 0.$$

$\sin\theta(2\sin\theta - 1) = 0$

$\Rightarrow \sin\theta = 0$ or $2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

$\Rightarrow \theta = 0, \pi, \frac{\pi}{6}, \frac{5\pi}{6}$

Answer: $\theta = 0, \frac{\pi}{6}, \frac{5\pi}{6}, \pi$

3. (8 points) Find a formula for the inverse $f^{-1}(x)$ where

$$f(x) = 10e^{x-1}.$$

Let $y = 10e^{x-1}$

Then $\frac{y}{10} = e^{x-1}$

$\Rightarrow \ln\left(\frac{y}{10}\right) = \ln(e^{x-1})$

$\Rightarrow \ln(y) - \ln(10) = x - 1$

$\Rightarrow x = \ln(y) - \ln(10) + 1$

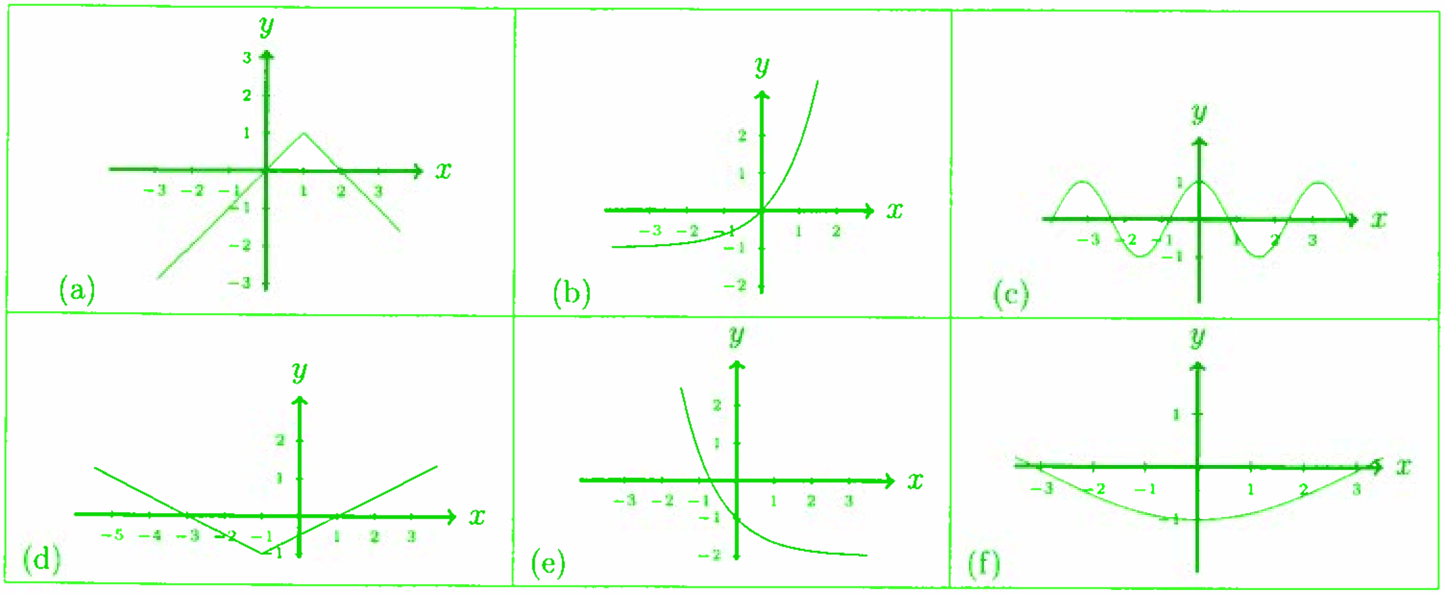
Therefore

$f^{-1}(x) = \ln(x) - \ln(10) + 1$

Answer: $f^{-1}(x) = \ln(x) - \ln(10) + 1$

4. (12 points) Match the following curves to their equations.

Curves:



Equations:

1.) $y = \|x\|$	5.) $y = e^{-x} - 2$	9.) $y = 2\cos x$
2.) $y = -2\|x-1\| - 1$	6.) $y = -e^{x} - 2$	10.) $y = \cos(2x)$
3.) $y = 1 - \|x-1\|$	7.) $y = e^{x} - 1$	11.) $y = -\cos(\frac{1}{2}x)$
4.) $y = \frac{1}{2}\|x+1\| - 1$	8.) $y = \ln x$	12.) $y = -\frac{1}{2}\cos(\frac{1}{2}x)$

Answers:

(a) 3 (b) 7 (c) 10 (d) 4 (e) 5 (f) 11

5. (10 points) Let $f(x) = \sqrt{3x+1}$ and $g(x) = \dfrac{1}{x}$.

(a) Find $(g \circ f)(x)$ and its domain.

$$(g \circ f)(x) = g(f(x)) = g(\sqrt{3x+1}) = \frac{1}{\sqrt{3x+1}}$$

This function is defined when $\sqrt{3x+1} \neq 0$ and $3x+1 \geq 0$

$\Longrightarrow 3x+1 > 0$

$\Longrightarrow 3x > -1$

$\Longrightarrow x > -\frac{1}{3}$.

Answer:

$(g \circ f)(x) = \dfrac{1}{\sqrt{3x+1}}$ has domain $\left(-\frac{1}{3}, \infty\right)$.

(b) Find $(f \circ g \circ f)(x)$.

$$(f \circ g \circ f)(x) = f(g(f(x))) = f(g(\sqrt{3x+1})) = f\left(\frac{1}{\sqrt{3x+1}}\right) = \sqrt{3 \cdot \frac{1}{\sqrt{3x+1}} + 1}$$

Answer:

$(f \circ g \circ f)(x) = \sqrt{\dfrac{3}{\sqrt{3x+1}} + 1}$

6. (8 points) Find the exact values.

(a) $\log_7(1)$ $= 0$ since $7^0 = 1$

Answer: 0

(b) $\log_{15}\left(\frac{1}{15}\right)$ $= -1$ since $15^{-1} = \frac{1}{15}$

Answer: -1

(c) $\log_3(100) + \log_3(0.81)$

$= \log_3(100 \cdot 0.81) = \log_3(81) = 4$

since $3^4 = 81$

Answer: 4

(d) $e^{-\ln(\ln e^4)}$

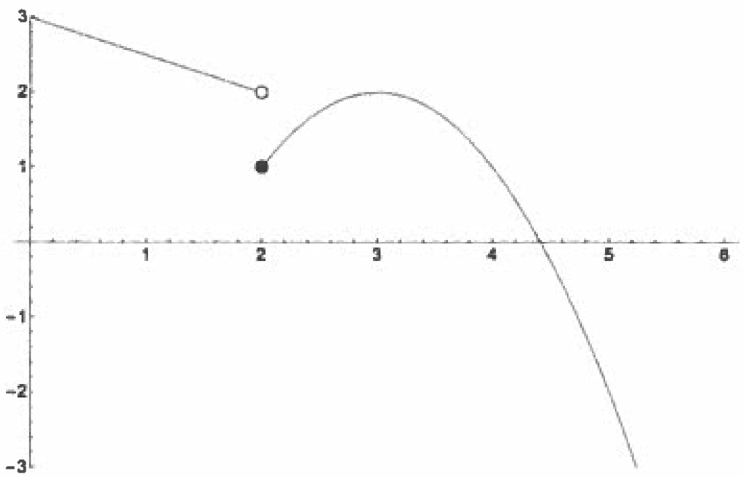
$= e^{-\ln(4)} = e^{\ln(4^{-1})} = 4^{-1}$

Answer: $\frac{1}{4}$

7. (10 points)

Deduce the following quantities from the corresponding graph. If the limit does not exist or the function is not defined, write **DNE**.

(i)



(a) $f(2) =$ 1

(b) $\lim_{x\to 2^-} f(x) =$ 2

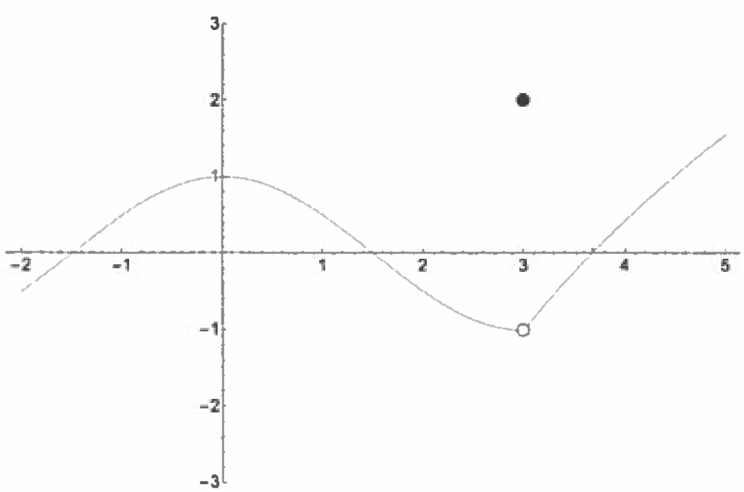
(c) $\lim_{x\to 2^+} f(x) =$ 1

(d) $\lim_{x\to 2} f(x) =$ DNE

Circle **T** for true or **F** for false:

The function is continuous at $x = 2$. **T** **F**

(ii)



(a) $g(3) =$ 2

(b) $\lim_{x\to 3^-} g(x) =$ −1

(c) $\lim_{x\to 3^+} g(x) =$ −1

(d) $\lim_{x\to 3} g(x) =$ −1

Circle **T** for true or **F** for false:

The function is continuous at $x = 3$. **T** **F**

8. (15 points) Find the following limits. Show all work. Simplify your answer.

If a limit is infinite, say whether it is $+\infty$ or $-\infty$.

(a) $\lim_{s\to 2} \frac{4s^2 - 3s}{\sqrt{3-s}}$

$$= \frac{4(2)^2 - 3(2)}{\sqrt{3-2}} = \frac{16-6}{\sqrt{1}} = 10$$

Answer: 10

(b) $\lim_{t\to -1} \frac{t^2 - 2t - 3}{t+1}$

$$= \lim_{t\to -1} \frac{\cancel{(t+1)}(t-3)}{\cancel{t+1}} = \lim_{t\to -1} (t-3) = -4$$

Answer: -4

(c) $\lim_{x\to 0} \tan^{-1}(e^x)$

$$= \tan^{-1}\left(\lim_{x\to 0} e^x\right) = \tan^{-1}(e^0) = \tan^{-1}(1) = \frac{\pi}{4}$$

Answer: $\frac{\pi}{4}$

(d) $\lim_{x \to 9} \frac{x+1}{(x-9)^2}$

- $\lim_{x \to 9^+} \frac{x+1}{(x-9)^2} = \infty$ since $x+1 \to 10$ and $(x-9)^2 \to 0$ as $x \to 9^+$ and $(x-9)^2 > 0$ for $x > 9$.

- $\lim_{x \to 9^-} \frac{x+1}{(x-9)^2} = \infty$ since $x+1 \to 10$ and $(x-9)^2 \to 0$ as $x \to 9^-$ and $(x-9)^2 > 0$ for $x < 9$.

Answer:

∞

(e) $\lim_{x \to 0} \frac{\sqrt{x+16} - 4}{x}$

$$= \lim_{x \to 0} \left(\frac{\sqrt{x+16} - 4}{x}\right)\left(\frac{\sqrt{x+16}+4}{\sqrt{x+16}+4}\right)$$

$$= \lim_{x \to 0} \frac{x+16-16}{x(\sqrt{x+16}+4)}$$

$$= \lim_{x \to 0} \frac{\cancel{x}}{\cancel{x}(\sqrt{x+16}+4)} = \lim_{x \to 0} \frac{1}{\sqrt{x+16}+4} = \frac{1}{\sqrt{16}+4} = \frac{1}{8}$$

Answer:

$\frac{1}{8}$

9. (9 points) Find all vertical asymptotes of the following function. Show all work and justify your answer.

$$f(x) = \frac{x+6}{x^2+5x-6}$$

$$f(x) = \frac{x+6}{(x+6)(x-1)}$$

so only possible vertical asymptotes are $x = -6, 1$.

- $\lim_{x\to -6} \frac{\cancel{x+6}}{\cancel{(x+6)}(x-1)} = \frac{1}{-6-1} = -\frac{1}{7}$ so $x=-6$ is NOT a vert. asymptote.

- $\lim_{x\to 1} f(x)$ does not exist since $\begin{cases} \lim_{x\to 1^+} f(x) = \lim_{x\to 1^+} \frac{1}{x-1} = \infty \\ \lim_{x\to 1^-} f(x) = \lim_{x\to 1^-} \frac{1}{x-1} = -\infty \end{cases}$

so $x = 1$ is a vertical asymptote.

Answer:

$x = 1$

10. (12 points) Let p and r be real numbers, and

$$h(x) = \begin{cases} 4x^2 + 1 & \text{if } x < 1 \\ 2 + p & \text{if } x = 1 \\ 3x^2 - 2x + r & \text{if } x > 1. \end{cases}$$

For which values of p and r is $h(x)$ continuous at $x = 1$? Justify your answer.

For $h(x)$ to be continuous at $x=1$, we need

$$\lim_{x\to 1} h(x) = h(1).$$

$$\left.\begin{array}{l} \bullet\ h(1) = 2+p \\ \bullet\ \lim_{x\to 1^-} h(x) = \lim_{x\to 1^-} (4x^2+1) = 5 \\ \bullet\ \lim_{x\to 1^+} h(x) = \lim_{x\to 1^+} (3x^2-2x+r) = 1+r \end{array}\right\}$$

So we need $2+p = 5 = 1+r$.

Therefore $p=3$ and $r=4$.

Answer:

$p=3, \quad r=4$

EXTRA PAGE. You may use this page if you run out of space. Be sure to label your problems on this page and also include a note on the original page telling the graders to look for your work here.